Aspiring for Change:  
A Theory of Middle Class Activism*

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July 3, 2014

Abstract. When there are two layers of uncertainty about the quality of a specific regime and governance in general, poor people perceive the current regime as bad, and rationally take this as an indication that all governments are bad. They believe that mass movements are futile for changing society and improving their welfare. The middle class are more sanguine about the prospect of good government. They have greater incentive to participate in collective political action, and expect more people to participate than the poor do. This coordination game admits interval equilibria but not monotone equilibrium. Only middle class people attack the regime and their action is successful only when the regime quality is intermediate.

Keywords. global game, model uncertainty, interval equilibrium, political passivity, abrupt outburst

JEL Classification. D74, D83, D84

*We thank Gabriel Carroll, Jimmy Chan, Stephen Chiu, Melody Lo, Stephen Morris, and seminar participants at 5th Shanghai Microeconomics Workshop, 2014 Asian Meeting of the Econometric Society, and University of Hong Kong for their suggestions. Yuxin Wang deserves praise for providing excellent research assistance to this project. Our research is partially funded by the Research Grants Council of Hong Kong (Project No. HKU 17500614).
1. Introduction

In recent years, the rise of the middle class in emerging and developing countries has been spectacular. The swelling ranks of the middle class are likely to produce fundamental impacts on economic and political developments in the countries concerned. In this paper, we focus on one of the salient aspects of this continuing global trend: “middle class activism.”

Middle class activism refers to the phenomenon that the middle class are among the most active stratum in society to exert pressure on governments to improve the quality of governance by participating in collective actions, such as protests or even political revolutions. The purpose of these mass actions is not necessarily to bring governments down. Rather, the common theme is often a demand for reform and a higher quality of governance, such as better public services and less corruption.

The political turmoil that swept Brazil, Bulgaria and Turkey in 2013 provide the most recent examples. These protests were stoked for distinct reasons in each country. However, the common fundamental strength of these demonstrations came from the involvement of the middle class. The phenomenon that middle classes take collective actions and demand changes in politics is anything but new in history. Both the 1987 demonstrations against Chun Doo-hwan in South Korea and the 1984 demonstrations against the Marcos regime in the Philippines were initiated and mobilized mostly by the urban middle class. More generally, in what is called the “third wave” of democratization between the 1970s and the early 1990s, the middle class were the most active participants and leaders, while the rich, the peasants or even the industrial workers remained relatively inactive or indifferent (Huntington 1993).

The World Values Survey provides some evidence that confirms a hump-shaped relationship between participation in collective political actions and the social class of participants. The survey asks respondents whether they have participated in demonstrations in the past, and to which social class they belong. We compute the fraction of participants in each class, after controlling for demographic characteristics of respon-

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1 The size of the middle class in the world is projected to increase from about 28 percent of the global population today to nearly two-thirds by 2030, outnumbering the poor for the first time in 2022 (e.g., RAND Europe 2013; Kharas and Gertz 2010).
2 The demonstrations in Turkey grew from a protest against plans for a construction project. An increase in public transport prices triggered the protests in Brazil. Corruption and cronyism in government set off the protests in Bulgaria. See “Middle-class rage sparks protest movement in Turkey, Brazil, Bulgaria and beyond,” Washington Post, June 29, 2013.
3 See “What happens when tear gas meets the middle class in Seoul?” The Economist, June 20, 1987, and Kimura (2003) for the role played by the middle class in these two countries.
4 The phenomenon of middle class activism is particularly relevant for non-democratic societies, where participating in mass actions is often the only way to produce political changes. However, middle class activism should not be equated to democratization, as the middle class do not necessarily embrace democratic values.
Figure 1. The lower class and the upper class are significantly less likely to participate in demonstrations than people in the middle social strata. Data source: The third, fourth and fifth waves of World Values Survey, covering 78 countries from 1994 to 2007.

It is interesting that the middle class, typically considered as the keystone of stability in a modern polity, are often spearheading social movements. Compared to the middle class, the rich have little incentive to upset the political system because they are its beneficiaries. But why are the poor less politically active than the middle class? If anything, they should be more dissatisfied with the current regime than the middle class are, and their opportunity costs of taking political action are lower. This puzzling fact of the poor being politically passive has long been known and documented by political scientists such as Huntington (1968). Berry, Portney and Thomson (1991) also find that political involvement is highly correlated with social class, and that the poor—who most need help from government—make the least demands of it. Hoffer (1951) observes that the poor “are not hospitable to change. [. . .] There is thus a conservatism of the destitute as profound as the conservatism of the privileged, and the former is as much a factor in the perpetuation of a social order as the latter.” These scholars typically take the passivity of the poor as a behavioral or psychological trait. In this paper, we seek to offer a rational explanation for why the poor are more “pessimistic” than the middle class, which accounts for the passivity of the former and the activism of the latter to bring about political changes.

Mass political actions are as much about hope as they are about discontent. To be motivated to take costly political action, individuals have to believe that the chance of success is sufficiently high, and that things will change for the better if they are successful. History is replete with examples of revolutions that turned sour. A case in point is the Orange Revolution of Ukraine in 2004–2005 which, despite successfully
ousting the incumbents, did not lead to the creation of an effective government. The country went through political turmoils after the revolution and eventually backslid into authoritarianism under President Yanukovych (Haran 2011). On the other hand, there are also instances when mass political actions altered the path of a country for the better—South Korea is a shining example.

We capture such uncertainty by postulating that people may adopt two alternative world views. Under the hopeful view, governments can be a force for good. The quality of governance is potentially high, even though the quality of the existing regime may not be high. Under the pessimistic view, the quality of governance is low in general, regardless of who is in power. If the existing regime is unsatisfactory, collective action—even when it succeeds—does not help because it would merely replace one set of corrupt leaders or inefficient policies by another. Individuals assess the plausibility of these two world views based on their personal experiences. The poor believe that the quality of the current regime is low. Because the quality of the current regime is an indicator of the quality of governance in general, the poor also rationally give more weight to the pessimistic world view. The middle class, on the other hand, attach a greater weight to the hopeful world view. Even though they do think that the existing government is of low quality (but not as bad as the poor believe it is), they are still optimistic that governance can be improved if the current leaders or their policies are replaced by a new draw.

Furthermore, such difference in world views between the middle class and the poor is reinforced by the difference in their assessments of the probability of success. We derive these probability assessments using a framework of coordination games, which is particularly useful for understanding mass political actions—one distinctive feature of the protests that we describe is that they are not organized by unions or other established political or interest groups, but are mostly spontaneous and leaderless. In this model, citizens decide whether to participate in a mass protest or not. If the number of participants is sufficiently high, the regime will be replaced (or reforms will be implemented); otherwise the regime and the old policies survive. Strategic complementarity implies that people are more likely to participate if they estimate that current economic conditions support a large middle class in society. Because they make inferences about economic conditions based on their own experience, people in the middle stratum believe that the size of the middle class is large. As a result, they are more optimistic about the chances of success than the poor are.

In equilibrium, the poor decide not to participate due to their pessimism. The rich

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5 Similarly in Egypt (although events are still unfolding there), the military establishment has regained much of its lost power since mass protests successfully removed President Mubarak in 2011.

6 See, for example, “The march of protest,” The Economist, June 29, 2013.
do not participate because they consider undesirable the regime change and the potential re-shuffling of economic status associated with it. The middle class, who see the possibility of better governance and who are optimistic about the chances of success, form the core participants of mass movements. This model therefore does not admit the standard monotone equilibrium in global games. Instead, the decision to participate in collective action is non-monotone in economic status; we label it an interval equilibrium. To our knowledge, there is little work in the global game literature that deals with interval equilibria with a continuum of agents. The method with which we establish equilibrium existence and multiplicity differs from the previous literature.

This model can also shed light on a number of aspects of middle class activism. For example, we show that the onset of a middle-class social movement can be abrupt. Society may appear tranquil and the only equilibrium is that no one attacks the regime. But a slight decrease in participation cost may open a “window of opportunity” for mass political action: the fraction of attackers in the population jumps discontinuously from zero to a strictly positive share. In addition, our model also predicts that the size and severity of mass protests need not be monotone in the quality of the current regime. It is possible, for example, that more people take to the streets to demonstrate as economic conditions improve.

The purpose of this paper is not to provide a comprehensive theory of mass movements. For example, our hypothesis may not be applied to explain revolutions in peasant or rural societies, where the vast majority of the population are poor farmers and the rest possess most of the wealth. Instead, we focus on mass movements in middle class societies, where the distribution of well-being or income is hump-shaped. However, our theory of political aspiration can be useful for explaining the low involvement of the poor and activism of the middle class in many other political activities.

Our paper does not address the link between education and participation in collective political actions. One popular idea about the observed middle class activism is that people in this social stratum are more educated than the poor, and educated individuals tend to be more politically involved. However, it is far from being settled that education critically affects political participation. For example, Friedman et al. (2011) show that education helps increase individuals’ political knowledge but does not lead to their participation in political activities. Kam and Palmer (2008) report that the effects of higher education on political participation disappear once pre-adult experiences and influences are taken into account. There is also a lack of evidence that education in less democratic countries helps promote political activism; to the contrary, some evidence suggests that schooling is one of the important vehicles for indoctrinating citizens (Lott 1999; Pritchett 2002). In any case, the mechanism proposed in our paper and the education nexus are not mutually exclusive.
2. Literature Review

We model mass collective action as a coordination game and adopt the regime change framework developed by Morris and Shin (1998). They show that in standard global games the equilibrium is monotone, and that the monotone equilibrium is unique. More recent developments of the literature have demonstrated that the uniqueness of equilibrium does not necessarily survive in richer set-ups (Angeletos and Werning 2006; Hellwig, Mukherji and Tsyvinski 2006). The most important difference of our work from the existing regime-change literature is that we characterize non-monotone “interval equilibria,” where citizens who receive intermediate signals take actions. The key to our equilibrium construction is that the net benefit of participating is non-monotone in welfare or economic status, which results in a non-monotone decision rule. It differs from one common feature of the previous literature (Angeletos, Hellwig and Pavan 2007; Bueno de Mesquita 2010), where the benefit of taking action for the “marginal attacker” is non-monotone. That feature leads to the equilibrium multiplicity in their models, but the decision rule still remains monotone.

Some implications of model uncertainty in global games have been explored by Chen and Suen (2014) in a dynamic setting, where both the underlying structure of the game and the fundamental strength of the regime are uncertain to agents. However, the focus of that paper is why crises that are rare tend to be contagious. Augmented with model uncertainty and dynamics, that model still features monotone and unique equilibrium, as in the standard global game literature. That is because participants always benefit from a successful attack, no matter which world view is the true description of how the world operates. In contrast, in this model, the assumption of model uncertainty leads to the existence of interval equilibria, because agents do not necessarily gain (that much) from a success, if the pessimistic world view is the true description.

Shadmehr and Bernhardt (2010) study a two-player coordination game with uncertain payoffs. In that paper, both cutoff and non-cutoff strategies can be adopted by one player, depending on the other players’ strategy. The symmetric non-cutoff strategy is obtained in their model because of strategic complementarity and the payoff structure. In our model, the game is played by a continuum of agents and cutoff equilibria do not exist. More importantly, the key to the existence of interval equilibria is that agents make Bayesian inference about the operation of the world under model uncertainty.

Substantively, our work contributes to the literature on the role of the middle class in modern societies. According to modernization theory (Lipset 1959), a growing middle class serves democracy, because this social stratum plays a mitigating role in moderating social conflicts and is less receptive to extremism. Barro (1999) offers evidence
that there may exist a positive relationship between the size of the middle class and the extent of democracy. Acemoglu and Robinson (2009) discuss the role of the middle class in the process of the creation and consolidation of democracy. In their framework, a large and affluent middle class may limit redistributive policies, which discourages the rich from repressing the poor. Our focus is not democratization in developing countries, but the phenomenon of middle class activism. Although both can be classified as political activities, demanding higher quality of governance and demanding democracy are conceptually distinct. As shown in the literature both empirically and theoretically, the middle class, although active in politics, do not necessarily embrace democracy (Huntington 1993; Robison and Goodman 1996; Chen and Lu 2011; Wang 2014). The key mechanism is also different. In our theory, the middle class aspire and fight for a better society, because they expect that such a goal is achievable through coordinated effort and that they can benefit from it.

Our idea that the poor tend to be politically passive is not entirely new. Both Huntington (1968) and Fukuyama (2013) argue that the poor do not care about politics because they are concerned mostly about day-to-day survival. However, that observation should be the result of the political passivity instead of the cause. If the poor believe that changes in politics brought about by mass movements can significantly improve their life quality, it would be most natural for them to participate, given the opportunity costs are low and the potential benefits are high. Fukuyama (2013) also argues that the failure of governments to meet the expectations of the middle class leads to their collective action against authorities, but does not explain why this social stratum, instead of the poor, hold such high expectations for the quality of governance. Our paper can be seen as an attempt to provide a rationale for the pessimism of the poor instead of just postulating it as a behavioral or psychological attribute. Such theoretical foundation is useful because it helps explain, among other things, why the size of the middle class affects their optimism, or why people who used to be politically quiescent when they were poor become active when their economic status improves.

3. The Model

We model mass collective action as a coordination game and adopt the global game framework of analysis. The model economy is populated by a continuum of agents indexed by \( i \in [0, 1] \). The well-being of agent \( i \) is \( x_i \), which can also be interpreted as his social status or his wealth or income. We assume that it is log-normally distributed, so that it is non-negative. Denote \( y_i = \log x_i \), and let \( y_i = \theta + \epsilon_i \). The first component \( \theta \) represents the “quality of governance,” or a regime-related aggregate factor that affects citizens’ welfare. Nature selects it from a normal distribution \( N(M, \sigma_\theta^2) \). The second component \( \epsilon_i \) is idiosyncratic and regime-independent, which is drawn from another
normal distribution $N(0, \sigma^2_\epsilon)$.

Citizens cannot observe the regime quality $\theta$, but only their own well-being $x_i$, which represents their own life experiences.

Each citizen $i$ chooses one of two actions: to participate in collective action ($a_i = 1$) or to not participate ($a_i = 0$). The aggregate mass of the population that join the protest is $A = \int_0^1 a_i \, di$. If the mass of participants is higher than a threshold $T \in (0, 1)$, the regime will be replaced by a new one or new reform policies will be in place; otherwise, the existing regime survives. A citizen incurs a positive cost $c$ if he participates. The perceived payoff from participation depends both on whether the regime survives and on whether he chooses to participate, as well as on the operation of the world.

There are two alternative models of the operation of the world. Under the hopeful world view, social movements can make a positive difference. This view holds that the quality of governance is potentially high, even though the quality of the existing regime may not be high. Therefore, when the current regime is not satisfactory, collective actions or pressure from society can either push the government to reform itself and to improve its quality, or replace the existing regime with a better one. The hopeful world view can be captured by two assumptions. On the aggregate level, if success is achieved, the replacement policy or regime is another independent draw $\theta'$ from the distribution $N(M, \sigma^2_\theta)$, with $M = m_H$.

On the individual level, the welfare consequence for agents is that they can experience a “re-shuffling” of economic status in the new society if their political action is successful. Specifically, with some probability, citizens can “start a new chapter in life,” and obtain a new status $x'_i$, where $\log x'_i = \theta' + \epsilon'_i$. This probability is $p_1$ for participants, and $p_0 < p_1$ for bystanders.

If the current regime survives, there will be no re-shuffling in society. The payoffs to agent $i$ under the hopeful view are summarized in Table 1.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$A \geq T$</th>
<th>$A &lt; T$</th>
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</thead>
<tbody>
<tr>
<td>$a_i = 0$</td>
<td>$p_0 x'_i + (1 - p_0) x_i$</td>
<td>$x_i$</td>
</tr>
<tr>
<td>$a_i = 1$</td>
<td>$p_1 x'_i + (1 - p_1) x_i - c$</td>
<td>$x_i - c$</td>
</tr>
</tbody>
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7This normality assumption is made so that the distribution of well-being in this economy is single-peaked, which captures one of the key features of modern middle class societies. It may not be applicable for rural societies where most of the population are poor farmers.

8Note that $\theta'$ can be either higher or lower than $\theta$. A higher $\theta'$ implies that the quality of the regime improves once the old one is replaced. If $\theta'$ is lower, then the mass action is successful but the quality of the new regime is even worse.

9In a model with a continuum of agents, agents have no incentive to participate if $p_1 = p_0$. The assumption that $p_1 > p_0$ reflects the fact that individuals care about the collective outcome as well as their personal role in bringing about the outcome, so that the free-riding problem does not completely eliminate participation.
There is also a pessimistic world view which holds that governments do not do any good regardless of who is in power. If the existing regime is unsatisfactory, changing it would merely replace one set of corrupt leaders or inefficient policies by another. According to this world view, society is in the grip of vested interests, which results in low quality of governance and any effort to improve it from the masses is in vain. Specifically, we assume that in this pessimistic world, \( M = \theta_L < \theta_H \) and the quality of the regime cannot be improved by social movements. As a result, there is no re-shuffling opportunity open to society. In other words, social movements are considered valueless for both society and individuals.

The key characteristics of the pessimistic world is that average regime quality is lower than that in the hopeful world. The assumption that the regime quality cannot be changed even upon a success simplifies the baseline model but is not crucial for our key results. In Section 6.1, we offer a generalized model in which we assume that a new draw of regime quality is allowed in each world upon a successful collective action, and explain why this assumption does not change our results.

Given the popularity of both world views in reality, it is realistic to assume that people are uncertain about which view is the true description of how the world works (see Jenkins and Klandermans 1995 and Santoro 2000). The hopeful view that social movements can bring about positive changes in governance has been widely documented and studied (Andrain and Apter 1995; Uba 2005). The pessimistic view that social protests cannot effectively solve the issue of low quality of governance is also popular. For example, Rochon and Mazmanian (1993) cast doubts on the usefulness of mass movements and argue that they often fail to achieve the goals initially sought; and Gamson (1990) suggests that mass protests rarely succeed in making broad structural changes in the political system.

In this model, three types of uncertainty matter for the decision of individuals—model uncertainty, fundamental uncertainty, and payoff uncertainty. First, agents are not certain about the true model of the world. We assume that they attach a common prior probability \( \alpha_0 \) that the hopeful view is true. Based on their personal experience, they rationally update their beliefs about the two world views. Since \( y_i = \theta + \epsilon_i \) and \( \theta \) is drawn either from \( N(\theta_H, \sigma^2_\theta) \) under the hopeful view or from \( N(\theta_L, \sigma^2_\theta) \) under the pessimistic view, the relative likelihood of observing \( y_i \) given these two world views is \( \phi((y_i - \theta_H)/\sigma_y) / \phi((y_i - \theta_L)/\sigma_y) \), where \( \phi(\cdot) \) is the standard normal density function and \( \sigma^2_y \equiv \sigma^2_\theta + \sigma^2_\epsilon \). Therefore, by Bayes’ rule, the updated probability of the hopeful world for agent \( i \) is:

\[
\alpha(y_i) = \frac{\alpha_0 \phi((y_i - \theta_H)/\sigma_y)}{\alpha_0 \phi((y_i - \theta_H)/\sigma_y) + (1 - \alpha_0) \phi((y_i - \theta_L)/\sigma_y)}.
\]

(1)
Second, the underlying regime quality $\theta$ is unknown to citizens. Since $\theta$ affects the distribution of $y_i$ in the population, it determines the total size of participants in the collective action, and hence its eventual success or failure. Because each agent forms a belief about the underlying regime quality with his own life experience, each agent also attaches a different probability that the collective action would succeed. We let

$$\pi(y_i) = \Pr[A(\theta) \geq T \mid y_i, H]$$

represent the probability that the current regime collapses conditional on the hopeful world view and on status index $y_i$. This probability depends on the equilibrium decision rule adopted by the agents.

Third, the payoff reward from the new society is not certain either. Conditional on the hopeful world, the re-shuffling of status following a successful mass action implies both upward and downward possibilities. For agent $i$, the expected gain of participation relative to standing by is:

$$\rho(y_i) = (p_1 - p_0)E[e^{\theta + \epsilon_i} - e^{y_i} \mid y_i, H] = (p_1 - p_0)(e^{\hat{y}} - e^{y_i}),$$

where $\hat{y} \equiv m_H + \sigma^2 y / 2$. When the free-riding problem is severe ($p_1 - p_0$ is small), the reward from successful mass action to an individual is diminished. Note that $\rho(y_i)$ may be positive or negative, depending on whether $y_i$ is less than or greater than $\hat{y}$.  

Equilibrium in this model is characterized by a participation set $Y^*$ and a success set $\Theta^*$, such that agent $i$ participates if and only if $y_i \in Y^*$, and the collective action is successful if and only if $\theta \in \Theta^*$. Since an agent participates in mass action when the expected benefit exceeds the cost, we require that for any $y_i \in Y^*$,

$$B(y_i; \Theta^*) \equiv a(y_i)\pi(y_i; \Theta^*)\rho(y_i) \geq c.$$

Since the mass action is successful when the size of participants exceeds $T$, we require that for any $\theta \in \Theta^*$,

$$A(\theta; Y^*) = \int_{y_i \in Y^*} \phi \left( \frac{y_i - \theta}{\sigma_e} \right) \frac{1}{\sigma_e} dy_i \geq T.$$

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\[10\] In the benchmark model, the expected benefit for agents with $y_i > \hat{y}$ is negative and they have a dominant strategy of not participating. This feature is not crucial for our characterization. In Section 6.1, we provide an extended model in which the expected benefit from participation is positive for all agents. More generally, we can also allow agents whose expected benefits are negative to take action and support the regime. We deal with this possibility implicitly in Section 6.3, where the mass of well-off agents is higher given better regime quality and the regime is less easy to overthrow.
4. Equilibrium Analysis

In this model, the expected reward $\rho(y_i)$ from successful mass action is decreasing in $y_i$. The rich find the potential re-shuffling in a new social order costly and have no incentive to upset the existing regime. Indeed, for any agent with $y_i > \hat{y}$, $\rho(y_i)$ is strictly negative, so he never participates. In contrast, poorer individuals desire social mobility and have relatively little to lose. They are the natural candidates to participate. We say that there is a monotone equilibrium if agents adopt a monotone decision rule; i.e., the equilibrium participation set $Y^*$ takes the form of a half interval, $(-\infty, \bar{y}]$, for some finite $\bar{y}$.

If there were no model uncertainty (i.e., everyone adopts the hopeful world view), it would be easy to see how a monotone equilibrium arises as an equilibrium outcome of this model. For any agent, the estimated chance of success $\pi(y_i)$ depends on the equilibrium construction. Consider a standard construction as follows. Agents participate when $y_i$ is below a threshold $\bar{y}$. As a result of this decision rule, the mass of participants decreases in $\theta$. Thus the regime is replaced if the regime quality $\theta$ is lower than a threshold $\bar{\theta}$. The poor believe that the regime quality is low, that a large fraction of the population also believe so, and that the mass of protesters must be high. This implies that both $\pi(y_i)$ and $\rho(y_i)$ are higher for poorer agents, which supports a monotone decision rule. However, this conclusion does not hold once we incorporate model uncertainty into the analysis.

Proposition 1. Monotone equilibrium does not exist in this model.

Proof. From equation (1), we have $\lim_{y_i \to -\infty} \alpha(y_i) = 0$, whereas both $\pi(y_i)$ and $\rho(y_i)$ are bounded above. Therefore, for $y_i$ sufficiently low, $B(y_i) < c$, which violates condition (4).

Individuals in this model are making two inferences based on $y_i$. They need to form expectations about the quality of the existing regime $\theta$, as well as the quality of governance in general, which is captured by the mean $M$ of the distribution, from which $\theta$ is drawn. Poorer agents infer that the current regime is bad (i.e., $\theta$ is low), but they also infer that all governments are bad (i.e., $M$ is low). The extreme pessimism of the very poor accounts for the failure to obtain a monotone equilibrium in this model. This mechanism is consistent with the classical view of Huntington (1968), who points out that the poor in underdeveloped countries “do not seriously expect their government to do anything to alleviate the situation.” We provide some further evidence that the poor take a pessimistic view of governance in general in Section 5.

In general, $\alpha(y_i)$ is increasing in $y_i$. This follows from the fact that the ratio of the likelihood of observing $y_i$ in the hopeful world relative to that in the pessimistic world,
\( \phi((y_i - m_H)/\sigma_y)/\phi((y_i - m_L)/\sigma_y) \) is increasing in \( y_i \). While the middle class are less dissatisfied with the present government because they believe that \( \theta \) is not so low, they are optimistic that their situation can be changed for the better through collective action because \( M \) is thought to be high. Since we have shown that individuals who are very poor or very rich do not participate in collective action, we look for interval equilibria, in which \( Y^* \) takes the form of an interval, \([y, \bar{y}]\), for finite \( y \leq \bar{y} \). A special case is a degenerate interval equilibrium, in which no one participates. This corresponds to the case \( y = \bar{y} \), which we can represent by \( Y^* = \emptyset \).

For any interval participation set \( Y = [y, \bar{y}] \), the mass of participants \( A(\theta; Y) \) is hump-shaped in \( \theta \). When \( \theta \) is low, most individuals have low \( y_i \); and when \( \theta \) is high, most individuals have high \( y_i \). In either case, the mass of agents with intermediate values of \( y_i \) is small. Let \( \Phi(\cdot) \) represent the cumulative standard normal distribution function. We have

\[
A(\theta; Y) = \Phi \left( \frac{\bar{y} - \theta}{\sigma_e} \right) - \Phi \left( \frac{\bar{y} - \theta}{\sigma_e} \right).
\]

This function reaches a maximum at \( \theta = (y + \bar{y})/2 \), and is strictly decreasing toward 0 as \( \theta \) goes to positive or negative infinity. Thus, the set of \( \theta \) for which \( A(\theta; Y) \geq T \) is either empty (if \( T \) is high) or an interval, \([\theta, \bar{\theta}]\), for some finite \( \theta \leq \bar{\theta} \) (if \( T \) is low). Therefore, if the equilibrium participation set \( Y^* \) is an interval, the equilibrium success set \( \Theta^* \) must also be an interval.

Fix any interval success set \( \Theta = [\theta, \bar{\theta}] \). The probability of success in equation (2) can be written as:

\[
\pi(y_i; \Theta) = \Phi \left( \frac{\bar{\theta} - \beta m_H - (1 - \beta) y_i}{\sqrt{\beta \sigma^2}} \right) - \Phi \left( \frac{\theta - \beta m_H - (1 - \beta) y_i}{\sqrt{\beta \sigma^2}} \right).
\]

In this formula, \( \beta m_H + (1 - \beta) y_i \) is the posterior mean of \( \theta \) given \( y_i \) and the hopeful view, and \( \beta \sigma^2 \) is the posterior variance, where \( \beta \equiv \sigma_e^2 / \sigma^2 \). It is straightforward to observe that \( \pi(y_i; \Theta) \) is hump-shaped in \( y_i \). The poor citizens at the bottom of society believe that the size of participants, or citizens with \( y_i \in [y, \bar{y}] \), is small; thus the probability of success is small. Likewise, the rich at the upper end of the spectrum also believe that there is not enough participants to successfully change the regime. Only the population from the intermediate segment attach a high probability to the event that \( \theta \in \Theta \). They are relatively more sanguine about the success of collective action.

In this model, individuals care not only about the probability of success but also what difference such a success would make to society and their life chances. Under this construction, the payoff from participation in collective action is low for poor agents because both \( \alpha(y_i) \) and \( \pi(y_i; \Theta) \) are low. The payoff is also low (or even negative) for rich agents because both \( \pi(y_i; \Theta) \) and \( \rho(y_i) \) are low. In other words, pes-
Figure 2. The benefit function $B(y; \Theta)$ is single-crossing and hump-shaped when positive, given the success set $\Theta$ is an interval. The size of attack $A(\cdot; Y)$ is hump-shaped, given the participation set $Y$ is an interval.

Simism and the perceived low chance of success override the potential gain from a new society and prevent the poor from participating. The perceived low chance of success and the concern about potential downward mobility discourage the well-off agents from upsetting the system. Only a group of middle class citizens actively engage in this movement—they are hopeful enough about the upcoming change in regime, sufficiently optimistic about the chance of success, and expect upward mobility in their individual life chances.

Lemma 1. Fix any non-empty interval success set $\Theta$. The expected benefit from participating, $B(y; \Theta)$, crosses zero once at $\hat{y}$ and from above, is hump-shaped in $y_i$ when $y_i < \hat{y}$, and approaches zero when $y_i$ approaches $-\infty$ or $\hat{y}$.

In Lemma 1, we prove (see the Appendix) that $\alpha(y_i)$ is increasing, $\pi(y_i; \Theta)$ is hump-shaped, and $\rho(y_i)$ is decreasing in $y_i$. Moreover, each of these functions is log-concave in $y_i$ when positive. Therefore, $B(y; \Theta)$ is increasing then decreasing for $y_i < \hat{y}$. This shows that individuals with $y_i$ in the intermediate segment of the population distribution have the greatest incentive to participate in collective action. See Figure 2(a) for an illustration.

A participation set $Y^* = [y, \overline{y}]$ and a success set $\Theta^* = [\theta, \overline{\theta}]$ constitute a (non-degenerate) interval equilibrium of the model if and only if

\begin{align*}
B(y; \Theta^*) &= B(\overline{y}; \Theta^*) = c; \\
A(\theta; Y^*) &= A(\overline{\theta}, Y^*) = T.
\end{align*}

Since both $B(\cdot; \Theta^*)$ and $A(\cdot; Y^*)$ are hump-shaped, conditions (7) and (8) imply equilibrium conditions (4) and (5). See Figure 2.
Let $\mathcal{I} = \{[y, \bar{y}] : y < \bar{y}\} \cup \emptyset$ be the set of all finite participation intervals (including degenerate ones). Consider the following mapping $f : \mathcal{I} \to \mathcal{I}$. For any participation interval $Y$, solve the equation $A(\theta; Y) = T$. If no solution exists or there is only one solution, assign $f(Y) = \emptyset$; when there are two solutions, label them $\underline{\theta}$ and $\overline{\theta}$ and let $\Theta = [\underline{\theta}, \overline{\theta}]$. For such $\Theta$, solve the equation $B(y_i; \Theta) = c$. If no solution exists or there is only one solution, assign $f(Y) = \emptyset$; otherwise label the two solutions $\underline{y}'$ and $\overline{y}'$ and assign $f(Y) = [\underline{y}', \overline{y}]$. An equilibrium participation interval $Y^*$ is a fixed point of $f$.

**Lemma 2.** The mapping $f$ is monotone according to the set-inclusion order, i.e., $Y_1 \supseteq Y_0$ implies $f(Y_1) \supseteq f(Y_0)$.

**Proof.** If $f(Y_0) = \emptyset$, then the result is trivially true. Assume $f(Y_0) \neq \emptyset$, and consider $Y_1 \supseteq Y_0$. It is obvious that $A(\theta; Y_1) \geq A(\theta; Y_0)$ for any $\theta$. Because $A(\theta; Y)$ is hump-shaped, an upward shift in this function lowers the smaller root and raises the larger root to the equation $A(\theta; Y) = T$. Denote the respective solutions to the equations $A(\theta; Y_1) = T$ and $A(\theta; Y_0) = T$ by $\Theta_1$ and $\Theta_0$. We have $\Theta_1 \supseteq \Theta_0$. A wider success interval raises the estimated probability of success for every agent. Thus, $B(y_i; \Theta_1) \geq B(y_i; \Theta_0)$ for any $y_i < \hat{y}$. Since $B(y_i; \Theta)$ is hump-shaped, the respective solutions to the equations $B(y_i; \Theta_1) = T$ and $B(y_i; \Theta_0) = T$, denoted $Y'_1$ and $Y'_0$, satisfy $Y'_1 \supseteq Y'_0$. 

Since the empty set $\emptyset$ is a fixed point of $f$, a degenerate interval equilibrium always exists in this model. If no one attacks the regime (the participation interval is an empty set), then $A(\theta; \emptyset) < T$ for any $\theta$, which implies that the set of successful states is empty. If the success interval is empty, then $B(y_i; \emptyset) < c$ for any $y_i$, so no one attacks.

When the cost of participation $c$ is sufficiently high, the equation $B(y_i; \Theta) = c$ has no solution for any $\Theta$, because the benefit from participation is bounded above. In that case, a degenerate equilibrium is the only equilibrium. For low values of $c$, a degenerate equilibrium is still an equilibrium, but there are non-degenerate interval equilibria as well.

**Proposition 2.** For any $T \in (0, 1)$, there is a critical value $\hat{c}(T)$ such that non-degenerate interval equilibria exist if and only if $c \leq \hat{c}(T)$.

**Proof.** Find a large enough finite participation interval $\bar{Y}$ such that the upper boundary of this interval is strictly less than $\hat{y}$ and $\max_{\theta} A(\theta; \bar{Y}) > T$. Such $\bar{Y}$ exists because $\max_{\theta} A(\theta; \bar{Y})$ goes to 1 as the lower boundary of $\bar{Y}$ goes to $-\infty$. Let $\bar{\Theta}$ represent the interval of $\theta$ for which $A(\theta; \bar{Y}) \geq T$. Pick a sufficiently small $c$ such that $B(y_i; \bar{\Theta}) > c$ for any $y_i \in \bar{Y}$. Such $c$ exists because $B(y_i; \bar{\Theta})$ is hump-shaped when $y_i < \hat{y}$ and approaches 0 when $y_i$ approaches $-\infty$ or $\hat{y}$. Recall that $f(\bar{Y})$ is the set of $y_i$ for which $B(y_i; \bar{\Theta}) > c$. By construction, we have $f(\bar{Y}) \supseteq \bar{Y}$. Let $Y_{\max}$ represent the interval of $y_i$.
for which \(a(y_i)\rho(y_i) \geq c\). Since \(a(y_i)\rho(y_i) > B(y_i; \Theta)\) for any finite success interval \(\Theta\), we have \(Y_{\text{max}} \supset f(Y)\) for any \(Y\). Therefore,

\[
Y_{\text{max}} \supset f(Y_{\text{max}}) \supset f(\hat{Y}) \supset \hat{Y}.
\]

Denote the restricted domain \(I^* = \{Y : Y_{\text{max}} \supset Y \supset \hat{Y}\}\). Any element of \(I^*\) is a non-degenerate interval. Moreover, the partially ordered set \((I^*, \supseteq)\) is a complete lattice, with supremum \(Y_{\text{max}}\) and infimum \(\hat{Y}\). Since both \(f(Y_{\text{max}})\) and \(f(\hat{Y})\) belong to \(I^*\), and since \(f\) is monotone, we have \(f(Y) \in I^*\) for any \(Y \in I^*\). In other words, \(f\) is a monotone mapping from \(I^*\) to \(I^*\). By Tarski’s fixed point theorem, a fixed point of \(f\) in \(I^*\) exists.

Next, we show that if a non-degenerate interval equilibrium exists for some \(\hat{c}\), then a non-degenerate interval equilibrium exists for any \(c < \hat{c}\). To see this, let \(\hat{Y}^*\) be an equilibrium participation interval when \(c = \hat{c}\). Since \(f(\hat{Y}^*; \hat{c}) = \hat{Y}^*\) and \(c < \hat{c}\), we must have \(f(\hat{Y}^*; c) \supset \hat{Y}^*\). Moreover we have already established that \(Y_{\text{max}} \supset f(Y_{\text{max}}; c)\). This means that for any \(c < \hat{c}\), \(f(\cdot; c)\) is a monotone mapping from \(I^*(c)\) to \(I^*(c)\), where \(I^*(c) = \{Y : Y_{\text{max}} \supset Y \supset \hat{Y}^*\}\). Tarski’s theorem guarantees the existence of a fixed point of \(f(\cdot; c)\).

Our approach to establishing the existence of interval equilibrium is different from that of Morris and Shin (1998). Their method of showing the existence of monotone equilibrium and eliminating non-monotone equilibria depends on the fact that the ranking of agents by their expected benefit from participating is always monotone and invariant to any monotone strategy. In contrast, in our model, the ranking of agents by their expected benefit is not monotone and is not invariant to their decision rule, because the \(B(y_i; \Theta)\) function depends on the success set, or how agents coordinate. Thus, there is no way to transform the ordering of agents into a one-dimensional object so that the monotone method of Morris and Shin (1998) can be applied.

Let \(Y^*\) be the largest equilibrium participation interval in the restricted domain \(I^*\), defined in the proof of Proposition 2. Note that it is also the largest one in the unrestricted domain \(I\). The smallest equilibrium in \(I\) is \(\emptyset\). To establish that multiple non-degenerate equilibria exist in this model, we construct another restricted domain \(I^{**}\) such that each element in \(I^{**}\) is a non-degenerate strict subset of \(Y^*\), and show that there exists another fixed point of \(f\) in \(I^{**}\).

**Proposition 3.** For any \(T \in (0, 1)\), multiple non-degenerate interval equilibria exist if \(c < \hat{c}(T)\).

In the proof of Proposition 3, we use the inverse mapping \(f^{-1} : I \to I\), defined as follows. Take any participation interval \(Y\). Find the success interval \(\Theta\) such that
\[ \{y_i : B(y_i; \Theta) \geq c\} = Y. \] From the \( \Theta \) so obtained, find the participation interval \( Y' \) such that \( \{\theta : A(\theta; Y') \geq T\} = \Theta. \) Assign \( f^{-1}(Y) = Y' \) if a non-degenerate solution exists in each step; otherwise assign \( f^{-1}(Y) = \emptyset. \) Note that \( f^{-1} \) is monotone according to the set-inclusion order.

The proof of Proposition 2 implies that there always exists some interval \( Y_m \) such that \( Y^* \supset Y_m \) and \( f(Y_m) \supset Y_m. \) Because \( f^{-1} \) is monotone, the latter is equivalent to \( Y_m \supset f^{-1}(Y_m). \) We can also show that there always exists some non-degenerate interval \( Y_l \) such that \( f^{-1}(Y_l) \supset Y_l. \) We choose \( Y_l \) such that:

\[ Y_m \supset f^{-1}(Y_m) \supset f^{-1}(Y_l) \supset Y_l. \]

This guarantees the existence of a fixed point of \( f^{-1} \) (and hence a fixed point of \( f \)) in \( \mathcal{I}^* = \{Y : Y_m \supset Y \supset Y_l\}. \)

Multiple equilibria exist because expectations are self-confirming. Many agents participate in a large equilibrium, producing a large probability of success, which in turn justifies their decision to participate. In a small equilibrium, many agents are bystanders because they do not expect success, which has the effect of confirming their initial expectations.

So far, our analysis has focused on interval equilibria. Recall that when the participation set is an interval, both the attack function \( A(\cdot; Y) \) and the associated benefit function \( B(\cdot; \Theta) \) are hump-shaped. The logic of strategic complementarity suggests that other forms of equilibria are possible. For example, suppose that the participation set \( Y \) consists of two non-overlapping intervals. Then, it is possible that the attack function \( A(\cdot; Y) \) may have two peaks (near the mid-points of the two participation intervals). A twin-peaked \( A(\cdot; Y) \) function may support a success set \( \Theta \) which consists of two non-overlapping intervals. Such a success set \( \Theta \) may in turn produce a \( B(\cdot; \Theta) \) that is twin-peaked in \( y_i, \) so that two non-overlapping intervals of agents indeed have the greatest incentive to participate. However, we note that even when such equilibria exist, our central insight that the very poor and the very rich do not actively take part in mass social movements still survives. Furthermore, the equilibrium with the largest set of participants must be an interval equilibrium and cannot be any other type.

**Proposition 4.** The participation set in any non-interval equilibrium is strictly contained in that of the largest interval equilibrium.

**Proof.** Since \( B(y_i; \Theta) \) approaches 0 for \( y_i \) very low and is negative for \( y_i > \hat{y} \), both the participation set and the success set must be unions of non-overlapping finite intervals. Suppose there is a non-interval equilibrium \( (Y^*_n, \Theta^*_n). \) Let \( Y^*_n = \bigcup_{j=1}^{J_1} [y_j^l, y_j^r] \) and \( \Theta^*_n = \bigcup_{j=1}^{J_2} [\theta_j^l, \theta_j^r], \) where \( J_1 \) and \( J_2 \) are positive integers and \( y_j^l < y_j^{l+1} \) and \( \theta_j^l < \theta_j^{l+1}. \)
Consider the participation interval $Y_0 = [y^1, y^1]$ and the success interval $\Theta_0 = [\theta^1, \theta^1]$. We have:

$$A(\theta^1; Y_0) > A(\theta^1; Y^*_n) = T,$$

$$A(\theta^1; Y_0) > A(\theta^1; Y^*_n) = T;$$

where the inequality follows from $Y_0 \supset Y^*_n$, and the equality follows because $\theta^1$ and $\theta^1$ are on the boundary of the equilibrium success set $\Theta^*_n$. Although $A(\theta; Y^*_n)$ may not be hump-shaped in $\theta$, $A(\theta; Y_0)$ is hump-shaped as $Y_0$ is an interval. Therefore, if $\Theta'$ is the set of $\theta$ for which $A(\theta; Y_0) \geq T$, the two inequalities above imply that $\Theta' \supset \Theta_0 \supset \Theta^*_n$. This, in turn, implies:

$$B(y^1; \Theta') > B(y^1; \Theta^*_n) = c;$$

$$B(\bar{y}^1; \Theta') > B(\bar{y}^1; \Theta^*_n) = c.$$

Recall that $f(Y_0)$ is given by the set of $y_i$ for which $B(y_i; \Theta') \geq c$. Therefore, we must have $f(Y_0) \supset Y_0$. As in the proof of Proposition 2, there exists at least one fixed point of $f$ in the domain $I^* = \{Y : Y_{\text{max}} \supset Y \supset Y_0\}$. Any fixed point in this domain is larger than $Y_0$, and is therefore larger than $Y^*_n$. $\blacksquare$

The largest interval equilibrium contains all other equilibria, either interval or non-interval. It provides an upper bound for the size of a protest and also implies that the very rich and the very poor, who are distributed outside the largest equilibrium participation interval, would never participate no matter how others act.

The largest participation interval $Y^*$ decreases in the participation cost $c$. A fall in participation cost encourages agents to take action and results in a larger participation interval. Because more agents participate in the mass action, the probability of success also increases in equilibrium. The equilibrium participation interval $Y^*$ approaches the half-interval $(-\infty, \hat{y}]$ when the cost $c$ approaches zero, but remains a finite interval for any positive $c$.

**Proposition 5.** In the largest equilibrium, both $Y^*$ and $\Theta^*$ decrease (according to the set-inclusion order) in $c$. Both sets have strictly positive measure at $c = \hat{c}(T)$ and change discontinuously to an empty set when $c$ exceeds $\hat{c}(T)$. Further, $\hat{c}(T)$ is decreasing in $T$.

Interestingly, the decision rule exhibits discontinuity: the largest equilibrium participation set shrinks (i.e., $y$ rises and $\bar{y}$ falls) as $c$ increases, until $c$ reaches the critical level $\hat{c}$, after which the interval equilibrium collapses and the degenerate equilibrium becomes the only equilibrium. See Figure 3 for illustration. It may be tempting to
conjecture that the equilibrium participation interval should shrink continuously to an empty set when the cost increases to the maximum of the expected benefit function. However, it is key to recognize that the benefit function $B(y_i; \Theta)$ depends on the success interval $\Theta$, which is endogenously determined by (8). When the participation interval $Y$ is small enough, the associated success interval $\Theta$ is so small that the expected benefit of participating $B(y_i; \Theta)$ is smaller than $c$ for any $y_i$. Therefore, the participation interval implodes to an empty set.

This discontinuity implies that a tiny reduction in participation cost may open a “window of opportunity” for mass collective action. When the cost of participation is slight above $\hat{c}$, agents cannot coordinate on an attack against the regime at all. However, when $c$ falls slightly below $\hat{c}$ for some reason, agents may coordinate either on the no-attack equilibrium or on an interval equilibrium. A random event can trigger the switch from one to another.\(^{11}\) In other words, there may not be attacks against the regime immediately following the reduction in cost, but a mass protest becomes possible. This feature of our model may help explain why protests often appear to be sudden outbursts and are massive in size when they emerge.

5. Evidence for Pessimism among the Poor

In this model, we rationalize the pessimism of the poor and the optimism of the middle class with Bayesian inference under model uncertainty, and further explain how these attitudes matter for their coordination in a collective action setting. The two linkages (from well-being to beliefs, and from beliefs to action) combined account for the phenomenon of middle class activism. This section offers some corroborating evidence for these two linkages.

The World Values Survey provides unique information about individuals’ beliefs

\(^{11}\)This implication of equilibrium multiplicity has been explored in related literature which features monotonic equilibria; see Angeletos, Hellwig and Pavan (2007), Bueno de Mesquita (2010), and Shadmehr and Bernhardt (2011).
or attitudes regarding governments and politics. In one set of questions, the survey describes four types of governments to respondents—having a strong leader, having experts make decisions, having the army rule, and having a democratic political system—and then asks what they think about each as a way of governing their country. If respondents consider one or more types of government good and the others bad, then this set of answers reveals their political preferences. However, there are people who dislike all the four types of government, which very likely means that they do not think governments can do any good and do not believe that their countries can be made better off by changing existing governments. Notably, this type of negative attitude towards governments and changes in politics is fairly close to the notion of pessimism in our model. Therefore, we regard a respondent to be pessimistic only if he views all the four types of governments as “very bad” or “fairly bad.”

In Table 2, we show regressions of the dummy variable for pessimistic on the income group of the respondent, using the lowest income quintile as the omitted category. Columns (1) and (2) of the table show that the coefficients for higher income quintiles are all negative and statistically significant, with or without controlling for personal characteristics (education, age, and sex). Moreover, the magnitude of the coefficients is larger for higher income quintiles. These results are consistent with the result that \( \alpha(y_i) \) is monotonic increasing in \( y_i \). Agents tend to be less pessimistic about governance in general when they enjoy higher welfare in society.

Columns (3) and (4) of Table 2 use an alternative dependent variable to capture the pessimistic world view. The World Values Survey asks respondents how interested they are in politics. We create a dummy variable uninterested, which takes the value of 1 if the response is “not very interested” or “not at all interested.” It is reasonable that people show little interest in politics if they hold a pessimistic view of governance in general and believe that changing governments cannot bring in real changes to society and their own lives. We also consider a stricter definition in columns (5) and (6), which requires both pessimistic and uninterested to be equal to 1. The results are similar to those in columns (1) and (2).

Furthermore, there is also evidence to suggest that people who hold more pessimistic world views about governance are less likely to participate in mass political actions. It is consistent with our second linkage which connects beliefs of individuals to their actions. To establish the relationship between the degree of pessimism and the likelihood of participation, we compute the fraction of participants among individuals who are pessimistic only, uninterested only, both pessimistic and uninterested, or neither, by using past participation in demonstrations as an indicator for political activism. The results are reported in Table 3, after controlling for demographic characteristics,
Table 2. Pessimistic View of Governance and Income Quintiles

<table>
<thead>
<tr>
<th>Income Quintile</th>
<th>pessimistic</th>
<th>uninterested</th>
<th>pessimistic × uninterested</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Lowest Quintile (1–20%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Second Quintile (21–40%)</td>
<td>−0.214*</td>
<td>−0.301**</td>
<td>−2.51***</td>
</tr>
<tr>
<td></td>
<td>(−2.08)</td>
<td>(−3.00)</td>
<td>(−8.27)</td>
</tr>
<tr>
<td>Third Quintile (41–60%)</td>
<td>−0.242*</td>
<td>−0.424***</td>
<td>−4.67***</td>
</tr>
<tr>
<td></td>
<td>(−2.22)</td>
<td>(−4.03)</td>
<td>(−14.35)</td>
</tr>
<tr>
<td>Fourth Quintile (61–80%)</td>
<td>−0.415**</td>
<td>−0.675***</td>
<td>−6.09***</td>
</tr>
<tr>
<td></td>
<td>(−3.28)</td>
<td>(−5.59)</td>
<td>(−15.92)</td>
</tr>
<tr>
<td>Highest Quintile (81–100%)</td>
<td>−0.467**</td>
<td>−0.856***</td>
<td>−7.52***</td>
</tr>
<tr>
<td></td>
<td>(−2.86)</td>
<td>(−5.54)</td>
<td>(−14.85)</td>
</tr>
</tbody>
</table>

Demographic characteristics include education, age, and sex. Levels of statistical significance are indicated by asterisks (* for 5%, ** for 1%, and *** for 0.1%), and t-statistics are in parentheses. Source: World Values Survey.

as well as country and wave fixed effects. The fraction of participants in each group is statistically significantly different from the others, with the most pessimistic group being least likely to participate in demonstrations. The results are similar if we do not control for demographics. This finding is in line with our model predictions, and further validates our constructed pessimism measures.

6. Extensions and Discussions

6.1. A generalized model

We make three specific assumptions in the baseline case: (1) there are only two alternative worlds; (2) in the pessimistic world, the regime quality cannot be changed even though the revolution is successful; and (3) the opportunity cost of participation depends on one’s economic status or well-being only in so far as successful collective action brings about a reshuffling of economic status. We show that the key results in our benchmark model still hold when we relax these assumptions.
Table 3. The Degree of Pessimism and Participation in Demonstrations

<table>
<thead>
<tr>
<th></th>
<th>pessimistic = 1</th>
<th>pessimistic = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>uninterested</td>
<td>9.08%</td>
<td>11.57%</td>
</tr>
<tr>
<td>uninterested</td>
<td>17.16%</td>
<td>21.54%</td>
</tr>
</tbody>
</table>

Data source: The third, fourth and fifth waves of World Values Survey, covering 80 countries from 1994 to 2007.

First, suppose that there are a continuum of possible world views, ranging from extremely pessimistic to extremely hopeful. Each world is described by a normal distribution $N(M, \sigma^2_M)$ from which the regime quality is drawn. Instead of assuming that the mean quality of governance $M$ is a two-point distribution with realization $m_H$ or $m_L$, assume that agents possess a common prior belief on the distribution of $M$, which is a normal distribution $N(\mu, \sigma^2_M)$, where $\mu$ is the mean of $M$ and $\sigma^2_M$ captures the degree of agents’ prior uncertainty over the possible world views. Each agent revises his belief over the possible world views based on his own experience. The posterior probability (density) that the world is $M = m$ is:

$$
\alpha(y, m) = \frac{1}{\sqrt{\gamma \sigma_m}} \phi \left( \frac{m - \gamma \mu - (1 - \gamma) y}{\sqrt{\gamma \sigma_m}} \right),
$$

where $\gamma = \sigma^2_y / (\sigma^2_y + \sigma^2_M)$.

Second, assume that in each world $m$, once the collective action succeeds, a new regime with quality $\theta'$ drawn from $N(m, \sigma^2_\theta)$ will replace the old one. All agents in the economy are affected by the change in regime, but participants receive an extra reward which is increasing in the new regime quality $\theta'$ if the revolt is successful. Specifically, the expected extra gain from participation conditional on world $m$ is:

$$
\pi(y, m)\rho(m) = \Pr[A(\theta) \geq T | y, M = m] r e^m,
$$

for some positive $r$. Note that, in contrast to the baseline model, a new draw of $\theta'$ is taken upon a successful collective action in all possible worlds, including those relatively pessimistic worlds (where average regime quality $m$ is relatively low). Since $\theta'$ centers around $m$, the expected extra gain would be lower in more pessimistic worlds and higher in more hopeful worlds. In those pessimistic worlds, successfully removing the status quo and replacing it with a new regime does not bring that much gain on expectation because the average of quality of governance is low.

Third, agents’ costs of participation may depend directly on their economic status. This may reflect considerations such as the time cost of taking action, or the income
loss resulting from reprisal by the regime. We let \( C(y_i) = c_0 + c_1 e^{y_i} \) represent the cost of participation for agent \( i \), where \( c_0 > 0 \) is a fixed cost component and \( c_1 e^{y_i} \) is a variable cost component, which is higher for individuals with higher income or wealth.

In this extended model, agents participate if and only if the expected benefit is higher than the cost:

\[
B(y_i) = \int_{-\infty}^{+\infty} \alpha(y_i, m) \pi(y_i, m) p(m) \, dm \geq C(y_i).
\]

**Proposition 6.** Given any finite success interval \( \Theta \), the benefit function \( B(y_i; \Theta) \) in the extended model is hump-shaped in \( y_i \), i.e., it increases from and then decreases towards 0. For any \( T \in (0, 1) \), there does not exist a monotone equilibrium, but interval equilibria exist for \( c_0 \) smaller than a critical value \( c_0^*(T) \).

The extended model does not admit a monotone equilibrium because agents at the bottom of society do not consider a successful collective action worthwhile. They tend to believe that those pessimistic worlds with low average regime quality are more likely to be the true world. Because the extra reward from participation in those worlds is expected to be low even if there is a new draw of regime quality, the expected benefit does not exceed the fixed cost \( c_0 \) of participation. For the very rich, even though they are optimistic about average regime quality and therefore expect positive returns from success, the regression-to-the-mean effect causes the benefit-cost ratio to go to zero as \( y_i \) goes to infinity. Thus, there is no monotone equilibrium in which the participation set takes the form \([y, \infty)\) either.\(^\text{12}\)

Interval equilibria exist in this model because we show that the benefit-cost ratio \( B(y_i; \Theta) / C(y_i) \) is hump-shaped in \( y_i \) for any success set \( \Theta \) that takes the form of an interval. It is the middle class who have the greatest incentive to participate in collective action. Moreover, in this extended model, the mapping \( f \) is also monotone. Therefore, our key results concerning the multiplicity of equilibria (Proposition 3), the comparison of interval and non-interval equilibria (Proposition 4), and comparative statics and discontinuity with respect to fixed cost (Proposition 5) are still valid.

### 6.2. Agents of different types

One implicit assumption in the benchmark case is that agents only differ in one dimension \( y_i \). As a consequence, agents from the lower and upper strata do not participate at all. In reality, participation in collective actions is hump-shaped in social class, as illustrated in Figure 1. Some poor or rich agents participate because of their special

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\(^\text{12}\)The regression-to-the-mean effect also affects the poor (to encourage them to participate) in this extended model. However, because their incomes or assets are so meager to begin with, this effect is not strong enough to offset the fixed cost of participation.
circumstances or their ideology. In general, people differ in traits that are unrelated to their social class, which may affect their subjective assessments of the value of revolutions for society and for themselves. We can capture this consideration by allowing for idiosyncratic propensity to participate.

Specifically, suppose that there are a continuum of types \(t_i\), representing some trait of agents which is independent of their well-being \(y_i\). Let \(B(y_i, t_i)\) be the expected benefit of participation for agent \(i\). Without loss of generality, we can assume that \(B(y_i, t_i)\) increases in \(t_i\) when \(B(y_i, t_i)\) is positive. Let \(g(t_i)\) be the density function of \(t_i\) and assume that this function is log-concave.

Proposition 7. Suppose \(B(y_i, t_i; \Theta)\) satisfies Lemma 1 for any \(t_i\) and is quasi-concave when positive for any success interval \(\Theta\). For any \(c < \hat{c}(T)\), there exist non-degenerate equilibria such that \(\Theta^*\) is an interval and \(Y^*\) is a convex set. For each \(t_i\), the set of \(y_i\) for which \((y_i, t_i) \in Y^*\) is an interval, and this interval expands as \(t_i\) increases. Further, the likelihood of participation is hump-shaped in \(y_i\).

The participation set \(Y^* = \{(y_i, t_i) : B(y_i, t_i; \Theta^*) \geq c\}\) is illustrated in Figure 4. The quasi-concavity assumption can be satisfied, for example, if \(B(y_i, t_i; \Theta) = t_i B(y_i; \Theta)\); it implies that the upper level set \(Y^*\) is convex. In Figure 4, agents with \(t_i\) lower than \(t_0\) do not participate regardless of his well-being \(y_i\). For agents of type \(t_0\), only those who receive \(y_0\) participate. That is, \(B(\cdot, t_0; \Theta^*)\) peaks at \(y_0\) and is equal to \(c\). For any \(t_i > t_0\), the set of \(y_i\) for which \(B(y_i, t_i; \Theta^*) \geq c\) is an interval, and this interval expands as \(t_i\) increases. In the proof of Proposition 7, we show that \(A(\theta; Y^*)\) is hump-shaped in \(\theta\) whenever \(Y^*\) is a convex set and \(g(\cdot)\) is log-concave. Thus, the equilibrium success set \(\Theta^*\) is an interval.

In this model, agents with very high or very low \(y_i\) may also participate, but only when they are very inclined to revolt, i.e., when \(t_i\) is very high. Other things equal, middle class agents with intermediate \(y_i\) expect greater benefits from participation; they join the protest even when their type \(t_i\) is not very high. Let \(\hat{t}(y_i)\) be the value of \(t_i\) that solves \(B(y_i, t_i; \Theta^*) = c\) for fixed \(y_i\). Figure 4 shows that \(\hat{t}(y_i)\) decreases then increases. Therefore, the likelihood of participation, \(Pr[t_i \geq \hat{t}(y_i)]\), increases then decreases in \(y_i\).

6.3. Alternative success-determination conditions

In the benchmark model, we assume that the collective action is successful if and only if the mass of attackers \(A(\theta)\) is higher than a threshold \(T\). A commonly used alternative assumption about the attack technology is that the regime collapses if and only if \(A(\theta) \geq \theta\). For example, it may be reasonable to assume that quality of the regime in
terms of the effectiveness of its governance is positively related to its ability to defend itself against mass attack. Note that this assumption implies that some regimes would collapse by themselves (when \( \theta \leq 0 \)) even when unprovoked by mass collective action. In this case, very poor citizens attach probability close to one that the regime would collapse, but they still have no incentive to participate because they take the pessimistic world view that governance cannot change for the better. In other words, a monotone equilibrium still does not exist with this alternative success-determination condition.

In an interval equilibrium under this alternative assumption, the participation set \( Y \) is an interval but the success set \( \Theta \) takes the form of a half-interval \((-\infty, \bar{\theta}]\) for some finite \( \bar{\theta} \). The \( \pi(y_i; \Theta) \) function is now strictly decreasing (instead of hump-shaped) when \( \Theta \) takes this form. But since \( a(y_i) \) is increasing, \( B(y_i; \Theta) \) is still hump-shaped and satisfies all the properties described in Lemma 1. This variation of the model highlights that model uncertainty, rather than the assumed attack technology, is the driving force for our key results. See Figure 5 for one such example.

One may also conjecture that whether success can be achieved often depends on some random aggregate factors such as luck, and that agents may not know exactly the threshold for success. We capture this consideration by adding an aggregate shock \( u \) to the success-determination condition, i.e., \( A(\theta, u) \leq T \). Assume that \( A(\theta, u; Y) \) is quasi-concave for any participation interval \( Y \), and that the density function \( h(u) \) is log-concave. Equilibrium in this case is characterized by a participation set \( Y^* \) which is an interval, and a success set \( \Theta^* = \{ (\theta, u) : A(\theta, u) \geq T \} \) which is a convex set. The proof of the existence of such interval equilibria is similar to that of Proposition 7.
6.4. Public news

This model can be extended to include public information. A noisy public signal can be interpreted in multiple fashion. For example, individuals may commonly have access to information regarding to the regime quality, i.e., news about how the regime functions. If the economic status in our model is interpreted as wealth or income, which is an indicator for their class identity, individuals may have some public knowledge or rough ideas about how wealth or income is distributed in society.

Assume that, in addition to $y_i$, agents receive a noisy public signal $s$ concerning the quality of the current regime, with $s = \theta + \zeta$. The common noise $\zeta$ is normally distributed with mean 0 and variance $\sigma^2_\zeta$, and is uncorrelated with either $\theta$ or $y_i$.

A public signal, as an independent source of information on the regime, affects the model in two aspects. First, a higher value of $s$ signals a higher quality of the regime. This causes agents to attach a higher weight to the hopeful world view. Its effect is similar to an increase in $\alpha_0$. In other words, on the arrival of a good news, everybody becomes more hopeful; that is, $\alpha(y_i; s)$ monotonically increases in $s$.

Second, the public news also affects the posterior belief of agents about the regime quality $\theta$ and therefore the likelihood of success. The likelihood $\pi(y_i; \Theta)$ may decrease or increase in response to an increase in $s$, depending on whether agent $i$’s posterior mean belief of regime quality is higher or lower than $(\bar{\theta} + \theta) / 2$. When $s$ is larger, the worst off agents tend to believe that success is more likely, because they think the quality of the regime $\theta$ has a higher chance to fall into the success interval $[\theta, \bar{\theta}]$. Agents on the top of society, on the other hand, already believe that $\theta$ is to the right of the success interval. Further increases in $s$ cause them to believe that success becomes even less likely. Thus, this effect tends to raise $B(y_i)$ for agents with low $y_i$ and reduce it for agents with high $y_i$. In other words, in response to a good news, the lower middle class are more inclined to take action and the upper middle class tend to drop out from

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{An example of interval equilibrium with an alternative success-determination condition. The participation set $Y^*$ is an interval but the success set $\Theta^*$ is a half-interval.}
\end{figure}
protests, because they think there are fewer participants.

While the net effect of these two mechanisms combined in general is ambiguous, our numerical results suggest that the first effect of updating world view can dominate the second, which results in an expansion of the equilibrium participation interval (\(y\) falls and \(\bar{y}\) rises). In other words, it is possible that more agents participate in collective action when they receive public news that their current regime is good.

Interestingly, when agents observe an extremely good public signal about regime quality, they are almost certain that the world is hopeful, i.e., \(\lim_{s \to \infty} a(y_i; s) = 1\). In this case, even the poorest agents are willing to participate in the protest. The lower bound of the equilibrium participation interval goes to negative infinity. Thus, once we remove uncertainty about the operation of the world, the interval equilibrium in this model approaches a monotone equilibrium.

7. Conclusion

In this paper, we rationalize the pessimism of the poor and the optimism of the middle class with model uncertainty, and provide some evidence which corroborates our model mechanism. More generally, the same mechanism can explain the passivity of the poor and activism of the middle class in many other political settings, such as voting, signing petitions, and working for a political party. For example, Jackson et al. (1998) empirically demonstrate that the poor are less inclined to vote in elections and more responsive to registration obstacles. It may be caused by the lack of desire and aspiration of lower class citizens, because they perceive the political system as distant and unresponsive to their interests.

Further, our work also shows how individuals with heterogeneous beliefs of how the world operates coordinate in a collective action. It enriches the literature of coordination games by developing some tools to analyze interval equilibria, beyond the standard monotone equilibria in this type of models. Our distinctive approach to establishing the existence and multiplicity of interval equilibria can be applied to coordination games with similar features.

A dynamic extension of our theory can potentially shed some light on the puzzling phenomenon that in less developed countries, the dissatisfaction of middle class often breaks out after the economic situation has actually improved, while in advanced countries, economic development reduces political instability (Russett et al. 1964; Huntington 1968). In our model, the equilibrium size of the attack on the regime and the equilibrium probability of success are both non-monotone in the regime quality. Our model predicts that the regime would not face a large scale of challenge when the regime quality is very low. In this case, the low quality of governance affects the
common component of the welfare of individuals and most of the agents are so pessimistic that they do not participate. The regime is most vulnerable when the quality of the regime is intermediate. When the well-being of agents in society is improved, a larger number of them fall into the participation interval and a large coordinated attack becomes more likely. In this model, this occurs when the regime quality is close to the midpoint of the equilibrium participation interval. When the quality of governance improves beyond that point, most of the individuals are generally content about their current status; they do not have incentive to change the status quo even though they are quite certain that they live in a hopeful world. Therefore, further enhancement of well-being can alleviate the possibility of political instability. However, being a static model, our model is not adequate to fully address how changing political and economic circumstances over time affect the evolution of beliefs about different world views. A dynamic analysis can be a fruitful direction for further research.
Appendix

Proof of Lemma 1. The function $B(y_i; \Theta)$ is single-crossing from above because $\rho(y_i) > 0$ if $y_i < \hat{y}$ and $\rho(y_i) < 0$ if $y_i > \hat{y}$. It goes to 0 when $y_i$ approaches $-\infty$ because $\lim_{y_i \to -\infty} a(y_i) = 0$; and it goes to 0 when $y_i$ approaches $\hat{y}$ because $B(\hat{y}; \Theta) = 0$ and $B(\cdot; \Theta)$ is continuous. Finally, it is straightforward to verify from equations (1) and (3) that $a(y_i)$ is log-concave and $\rho(y_i)$ is log-concave when positive. For $\pi(y_i; \Theta)$, we rewrite equation (6) as:

$$
\pi(y_i; \Theta) = \int_{\beta[m_H]}^{-\infty} \phi\left(\frac{t - 1 - \beta}{\sqrt{\beta \sigma_0}} y\right) \, dt \\
= \int_{-\infty}^{+\infty} \frac{1 - \beta}{\sqrt{\beta \sigma_0}} \phi\left(\frac{1 - \beta}{\sqrt{\beta \sigma_0}} (y - t)\right) I(t) \, dt,
$$

where the indicator function $I(t)$ is equal to 1 if $t \in ((\theta - \beta m_H) / (1 - \beta), (\theta - \beta m_H) / (1 - \beta)]$ and is equal to 0 otherwise. Since $\phi(\cdot)$ and $I(\cdot)$ are both log-concave, their convolution is log-concave (Prekopa-Leindler inequality) as well. Therefore, $B(y_i; \Theta) = \alpha(y_i) \pi(y_i; \Theta) \rho(y_i)$ is log-concave when positive, which implies that it is increasing then decreasing for $y < \hat{y}$.

Proof of Proposition 3. Observe that $a(y_i)\rho(y_i)$ is hump-shaped in $y_i$ and attains a maximum at some $y^p$, and that $\pi(y_i; [y_i - \omega, y_i + \omega])$ is also hump-shaped in $y_i$ and attains a maximum at $m_H$ for any $\omega > 0$. Define $\tilde{B}(y_i, \omega) \equiv B(y_i; [y_i - \omega, y_i + \omega])$. For any $\omega > 0$, this function is hump-shaped in $y_i$ and attains a maximum at some $y^*(\omega)$, which is between $y^p$ and $m_H$. The function $\tilde{B}(y^*(\omega), \omega)$ is strictly increasing in $\omega$ and approaches 0 when $\omega$ approaches 0.

For any $c \leq \max_{y_i} a(y_i)\rho(y_i)$, there exists an $\omega$ such that $\tilde{B}(y^*(\omega), \omega) = c$. Let $y^B \equiv y^*(\omega)$. We pick $Y_\epsilon = [y^B - \epsilon, y^B + \epsilon]$ for some small $\epsilon > 0$. Let $\Theta_\epsilon$ satisfy $\{y_i : B(y_i; \Theta_\epsilon) \geq c\} = Y_\epsilon$. By construction, $\Theta_\epsilon$ contains the interval $[y^B - \omega, y^B + \omega]$ and is arbitrarily close to this interval when $\epsilon$ approaches 0. Therefore, the mid-point of $\Theta_\epsilon$ can be made arbitrarily close to $y^B$. Let $Y'$ satisfy $\{\theta : A(\theta; Y') \geq T\} = \Theta_\epsilon$. Since $A(\cdot; Y')$ is symmetric about the mid-point of the interval $Y'$, the mid-point of $Y'$ and the mid-point of $\Theta_\epsilon$ must be the same. Furthermore, for any $T \in (0, 1)$, $Y'$ must have a strictly positive measure. Because the mid-points of $Y'$ and $Y_\epsilon$ are arbitrarily close to one another, and the measure of $Y_\epsilon$ can be made arbitrarily small, we have $f^{-1}(Y_\epsilon) = Y' \supset Y_\epsilon$ for $\epsilon$ sufficiently small.

For $\epsilon$ sufficiently small, we can always find a $\tilde{Y}$ such that it contains the interval between $y^p$ and $m_H$ and $f(\tilde{Y}) \supset \tilde{Y}$. That is because $m_H$ is always smaller than $\hat{y}$ and $f(\tilde{Y})$ is arbitrarily close to the half interval $(-\infty, \hat{y})$ when $c$ is very small. Then such a
\( \hat{Y} \) will satisfy \( \hat{Y} \supseteq Y_\varepsilon \) for small \( \varepsilon \). Therefore,

\[
\hat{Y} \supset f^{-1}(\hat{Y}) \supset f^{-1}(Y_\varepsilon) \supset Y_\varepsilon.
\]

Denote the set of intervals \( \mathcal{I}^* = \{ Y : \hat{Y} \supseteq Y \supseteq Y_\varepsilon \} \). Any element of \( \mathcal{I}^* \) is a non-degenerate interval. Moreover, the partially ordered set \( (\mathcal{I}^*, \supseteq) \) is a complete lattice, with supremum \( \hat{Y} \) and infimum \( Y_\varepsilon \). Since both \( f^{-1}(\hat{Y}) \) and \( f^{-1}(Y_\varepsilon) \) belong to \( \mathcal{I}^* \), and since \( f^{-1} \) is monotone, we have \( f^{-1}(Y) \in \mathcal{I}^* \) for any \( Y \in \mathcal{I}^* \). In other words, \( f^{-1} \) is a monotone mapping from \( \mathcal{I}^* \) to \( \mathcal{I}^* \). By Tarski’s fixed point theorem, a fixed point of \( f^{-1} \) in \( \mathcal{I}^* \) exists.

Together with Proposition 2, this argument implies that there exists some small \( \xi \) such that multiple non-degenerate equilibria exist for \( c \leq c' \). Let \( Y^* \) denote the largest non-degenerate equilibrium participation interval and \( Y^{**} \) denote a smaller one when \( c = c' \). By definition, \( Y^* \supset Y^{**} \).

Let \( Y_l = f(Y^{**}; c' + \eta) \). For small \( \eta \), \( Y_l \) is non-degenerate. Because \( f(Y; c) \) is decreasing in \( c \) for any \( Y \), we have \( f(Y^{**}; c') = Y^{**} \supseteq Y_l = f(Y^{**}; c' + \eta) \). Let \( Y_m \) denote the largest equilibrium participation interval when \( c = c' + \eta + \xi < \hat{c} \), where \( \xi > 0 \). We have \( f(Y_m; c' + \eta) \supseteq Y_m = f(Y_m; c' + \eta + \xi) \). For \( \eta + \xi \) sufficiently small, \( Y_m \) is sufficiently close to \( Y^* \) and thus \( Y_m \supset Y^{**} \supset Y_l \). Therefore,

\[
Y_m \supset f^{-1}(Y_m; c' + \eta) \supset f^{-1}(Y_l; c' + \eta) \supset Y_l.
\]

Using a similar logic as before, a fixed point of \( f^{-1}(\cdot; c' + \eta) \) exists in \( \mathcal{I}^{**} = \{ Y : Y_m \supseteq Y \supseteq Y_l \} \). Moreover, the proof of Proposition 2 establishes a fixed point of \( f(\cdot; c' + \eta) \) exists in \( \mathcal{I}^* = \{ Y : Y_{\text{max}} \supseteq Y \supseteq Y_m \} \). Therefore, \( f \) has at least two non-degenerate fixed points for any \( c \leq c' + \eta \).

Proceeding iteratively by replacing \( c' + \eta \) for \( c' \), this argument can be repeated for any \( c' < \hat{c} \). Thus, \( f \) has at least two non-degenerate fixed points whenever \( c < \hat{c} \).

**Proof of Proposition 5.** Let \((Y_1, \Theta_1)\) and \((Y_2, \Theta_2)\) be the largest equilibria when the costs are \( c_1 \) and \( c_2 \), respectively, with \( c_1 < c_2 < \hat{c} \). Since \( B(y_i; \Theta_2) - c \) strictly decreases in \( c \) for any \( y_i \), we have \( f(Y_2; c_1) \supset f(Y_2; c_2) = Y_2 \). Thus, \( f(\cdot; c_1) \) has a fixed point in the set \( \mathcal{I}^* = \{ Y : Y_{\text{max}} \supseteq Y \supseteq Y_1 \} \), which implies that \( Y_1 \supset Y_2 \). Furthermore, since \( A(\hat{\theta}; Y_1) > A(\hat{\theta}; Y_2) \) for any \( \hat{\theta} \), we have \( \Theta_1 \supset \Theta_2 \).

The second part of the proposition follows from the fact that \( f(Y; T) \) is decreasing in \( T \) for any \( Y \). Let \( \hat{Y}^* \) be a fixed point of \( f \) when the threshold is \( T \) and when \( c = \hat{c}(T) \). Then, for a lower threshold \( T' < T \), we have \( f(\hat{Y}^*; T') \supset \hat{Y}^* \). A fixed point of \( f(\cdot; T') \) exists in \( \mathcal{I}^* = \{ Y : Y_{\text{max}} \supseteq Y \supseteq \hat{Y}^* \} \) when the threshold is \( T' \) and when \( c = \hat{c}(T) \). This
implies \( \hat{c}(T') \geq \hat{c}(T) \).

**Proof of Proposition 6.** Since \( \alpha(y_i, m) \) is the density function of a normal distribution with mean \( \gamma \mu + (1 - \gamma)y_i \) and variance \( \gamma \sigma_m^2 \), we have:

\[
\alpha(y_i, m) \rho(m) = r e^{\gamma \mu + (1 - \gamma)y_i + \gamma \sigma_m^2/2} \frac{1}{\sqrt{2\pi\sigma_m}} \phi \left( \frac{m - (\gamma \mu + (1 - \gamma)y_i + \gamma \sigma_m^2)}{\sqrt{\gamma \sigma_m}} \right),
\]

which also takes the form of a normal density. Therefore, for any \( \theta_0 \),

\[
\int_{-\infty}^{\infty} \alpha(y_i, m) \rho(m) \Pr[\theta < \theta_0 \mid y_i, M = m] \, dm = r e^{\gamma \mu + (1 - \gamma)y_i + \gamma \sigma_m^2/2} \Phi \left( \frac{\theta_0 - \beta \gamma \mu - (1 - \beta \gamma)y_i - \beta \gamma \sigma_m^2}{\sqrt{\beta \sigma^2 + \beta^2 \gamma \sigma_m^2}} \right),
\]

where we make use of the fact that \( E[\Phi(ax + b)] = \Phi(b/\sqrt{1 + a^2}) \) when \( X \) is a standard normal random variable. This expression goes to 0 as \( y_i \) goes to negative infinity, while \( C(y_i) \) goes to \( c_0 > 0 \) as \( y_i \) goes to negative infinity. Therefore, the very poor never participates. There is no monotone equilibrium in the extended model.

Take any interval success set \( \Theta = [\underline{\theta}, \overline{\theta}] \). The benefit function \( B(y_i; \Theta) \) is equal to:

\[
re^{\gamma \mu + (1 - \gamma)y_i + \gamma \sigma_m^2/2} \times \left[ \Phi \left( \frac{\overline{\theta} - \beta \gamma \mu - (1 - \beta \gamma)y_i - \beta \gamma \sigma_m^2}{\sqrt{\beta \sigma^2 + \beta^2 \gamma \sigma_m^2}} \right) - \Phi \left( \frac{\underline{\theta} - \beta \gamma \mu - (1 - \beta \gamma)y_i - \beta \gamma \sigma_m^2}{\sqrt{\beta \sigma^2 + \beta^2 \gamma \sigma_m^2}} \right) \right].
\]

This function is log-concave in \( y_i \), and goes to 0 as \( y_i \) approaches positive or negative infinity. The cost function \( C(y_i) \), on the other hand, is log-convex, and is bounded away from 0. This implies that the function \( B(y_i; \Theta)/C(y_i) \) is increasing then decreasing. If \( c_0 \) is greater than some critical value \( \bar{c}_0 \), the equation \( B(y_i; \Theta)/C(y_i) - 1 = 0 \) has no solution; if \( c_0 \) is lower than that critical value, it has two solutions. The proof of the existence of an interval equilibrium for \( c_0 \leq \bar{c}_0 \) follows the same steps as in Proposition 2.

**Proof of Proposition 7.** The key to the proof is to show that there exist equilibria in which \( \Theta^* \) is a non-empty interval. Consider any success interval \( \Theta \). Since \( B(\cdot; \Theta) \) is quasi-concave, the participation set \( Y = \{ (y_i, t_i) : B(y_i, t_i; \Theta) \geq c \} \) is convex. Let \( \hat{t}(y_i) \) be the minimum \( t_i \) that satisfies \( B(y_i, t_i; \Theta) \geq c \). If no such \( t_i \) exists, define \( \hat{t}(y_i) \) to be the highest \( t_i \) in the support. Since the level set is convex, the function \( \hat{t} \) is convex.
Given the participation set $Y$, the mass of attackers can be written as:

$$A(\theta; Y) = \int_{-\infty}^{\infty} [1 - G(\hat{t}(y_i))] \phi\left(\frac{y_i - \theta}{\sigma_c}\right) \frac{1}{\sigma_c} \, dy_i,$$

where $G(t_i)$ is the cumulative distribution function of $t_i$.

To show that $A(\theta; Y)$ is hump-shaped, the following two facts are sufficient. First, $A(\theta; Y)$ approaches 0, when $\theta$ goes to $+\infty$ and $-\infty$. Second, $A(\theta; Y)$ is log-concave, which follows from the log-concavity of $1 - G(\hat{t}(y_i))$ and $\phi(\cdot)$. To see this,

$$\frac{d^2 \log[1 - G(\hat{t}(y_i))] }{dy_i^2} = -\left(\frac{g}{1 - G}\right)' \left(\frac{d\hat{t}}{dy_i}\right)^2 - \frac{g}{1 - G} \frac{d^2 \hat{t}}{dy_i^2} < 0.$$

This inequality holds because the hazard rate is increasing when $g(\cdot)$ is log-concave and because $\hat{t}(\cdot)$ is convex. Therefore, we may construct a mapping which takes an interval $\Theta$ and solve for the convex set $Y$ that satisfies $B(y_i, t_i; \Theta^*) \geq c$, and from such $Y$ solve for the interval $\Theta'$ that satisfies $A(\theta; Y) \geq T$. Following the same logic as the proof of Proposition 2, we can show that a fixed point of this mapping exists for $c$ less than some critical value $\hat{c}$. Further, if $t' > t$, $B(y_i, t'; \Theta^*) > B(y_i, t; \Theta^*)$ for each $y_i$ such that $B(y_i, t; \Theta^*)$ is positive. Therefore, $(y_i, t) \in Y^*$ implies $(y_i, t') \in Y^*$.

The likelihood of participation for each $y_i$ is the probability that $t_i$ is higher than $\hat{t}(y_i)$; that is, $\Pr[t_i > \hat{t}(y_i)]$. Since $B(y_i, t_i; \Theta^*)$ is strictly increasing in $t_i$, there exists a $t_0$ such that $\max_{y_i} B(y_i, t_0; \Theta^*) = c$. Let $y_0 = \arg\max_{y_i} B(y_i, t_0; \Theta^*)$. Since $B(y_i, t_i; \Theta^*)$ is hump-shaped, $\hat{t}(y_i)$ is decreasing for $y_i < y_0$ and increasing for $y_i > y_0$. Therefore,

$$\frac{d \Pr[t_i > \hat{t}(y_i)]}{dy_i} = -g(\hat{t}(y_i)) \frac{d\hat{t}(y_i)}{dy_i}$$

is positive for $y_i < y_0$ and negative for $y_i > y_0$. 

\[\square\]
References


